COUPLED BUCKLING DESIGN OF STEEL LACED COLUMNS

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A new design method developed by the authors that gives an opportunity to account for the interaction of different modes of buckling is presented. Eurocode 3 model is used as the reference model. In the reference model, an equivalent geometric imperfection comprising of an initial bow being equal to the length divided by 500 is used. The model is based on the design criterion referred to the individual chord components. It allows to avoid the application of a complicated procedure based on an explicitly given imperfection parameter. It is achieved by a treatment of the chord local buckling mode between lacing connections in the same way as the plate buckling of chord sectional elements. The buckling design of the compound member can therefore be carried out in the same way as for a single element provided that the shear stiffness of lacing members is properly taken care of.

Keywords: *Laced column, coupled buckling, design concept, implicit imperfection parameter model*

1 INTRODUCTION

Structural compression elements, such as columns supporting civil and mining engineering structures, are frequently made of several components (chords) put apart and tied up with use of plates (battens) or manufactured profiles (lacing members). Thus the former are called battened columns while latter – laced columns. When designing compound columns, the engineer has to include the interaction of all possible local and global instability modes. Usually the effects of local instability of chord sectional segments and flexural buckling of the column as a whole are those accounted for in design. The effect of buckling of individual components is usually checked separately.

Figure 1: Example of a laced column in mineral industry

Fig. 1 shows the example of a steel laced column in mineral industry. Compound columns support a transportation pipeline system and they are typical elements of supporting structures in mining industry. When designing such columns, the engineer has to include the interaction of all possible local and global instability modes. Usually the effects of local instability of chord sectional segments and flexural buckling of the column as a whole are those accounted for in design. The effect of buckling of individual components is usually checked separately. The recent code ENV 1993-1-1 [1] has introduced a method that is referred here as to the method of explicit imperfection parameter (EIP model). Built-up compression members consisting of two or more main components connected together at intervals to form a single compound member are therefore designed by the incorporation of an equivalent geometric imperfection. The effect of deformation of the compound member is taken into account by an amplification of the first order internal forces and moments in the main components, internal connection and any subsidiary components such as lacings. The method is rather complicated when used in design offices, especially when structural components of compound members are subjected to coupled instability (local and global buckling).

A convenient alternative method to be used for design of steel compound members is proposed by the authors. That method called here the method of implicit imperfection parameter (IIP model) is based on the concept introduced to the Polish national steel design code [2]. In the proposed method, the local buckling of chord sectional walls and the buckling of chord components between lacing connections are treated as local modes of failure of compound members, therefore they are assumed to affect (to reduce) the cross sectional resistance of the compound member. The reduced sectional resistance is then used for the buckling design of compound member. The compound member is treated as a member with finite flexural and shear stiffnesses [3]. The shear stiffness of the lacings is taken into account in the evaluation of the compound member slenderness ratio. The method is rather simple and allows for design of compound members in the same way as single component members.

2 EUROCODE'S METHOD OF EXPLICIT IMPERFECTION PARAMETER (EIP MODEL)

The method of explicit imperfection parameter is used in Eurocode 3 for design of compound columns (laced or battened). An equivalent geometric imperfection comprising of an initial bow not less than $e_0=l/500$ is recommended to account for any deviation of the real member stability model (Fig. 2a) from the perfect member stability model (Fig. 2b).

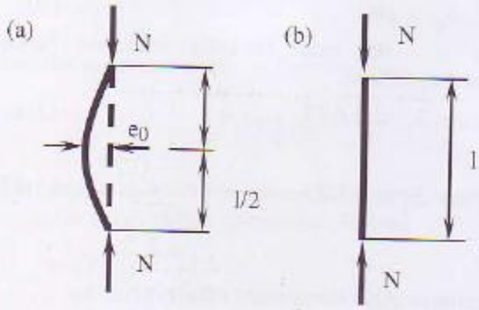


Figure 2: Basic models of compression members: a) imperfect model, b) perfect model

Referring to Fig. 2a, the differential equilibrium equation of the imperfect compound compression member can be written in the following format:

$$EI_{red}v^{IV} - N_{Sd}(v+v_0)^{II} = 0 \qquad (1)$$

where the reduced member flexural rigidity:

$$EI_{red} = EI\left(1 - \frac{N_{Sd}}{S_v}\right) \qquad (2)$$

and: E - Young modulus, I - cross section moment of inertia in the plane of initial bow e_0, N_{Sd} - design value of the member load (member compressive force), S_v- shear stiffness of the lacing system (the shear force required to produce unit shear deformation), v- displacement of an arbitrary point of the member longitudinal axis, v_0 - initial displacement due to the member crookedness, and finally superscripts IV and II are the symbols describing the 4th and the 2nd derivative of the displacement variables, respectively.

Assuming that the initial displacements constitute the sinus wave with the maximum coordinate of $e_0=l/500$, the solution of equation (1) can be written as follows:

$$v = v_{max}\sin\left(\frac{\pi z}{l}\right) \qquad (3)$$

where: v_{max}- second order displacement at the mid-length of the compound member that yields from a simple amplification rule:

$$v_{max} = \frac{e_0}{1 - \dfrac{N_{Sd}}{N_{cr,red}}} \qquad (4)$$

z - coordinate measured along the member length, $N_{cr,red}$- Timoshenko critical load of the perfect compound member (flexural buckling load taking into account the effect of finite shear deformations of the chord lacing system:

$$N_{cr,red} = \left(\frac{1}{N_{cr}} + \frac{1}{S_v}\right)^{-1} \qquad (5)$$

N_{cr} - Euler load of the perfect compound member:

$$N_{cr} = \frac{\pi^2 EI_{eff}}{l^2} \quad \text{or}$$

$$N_{cr} = \frac{\pi^2 EA}{\lambda_{eff}^{\ 2}} \qquad (6)$$

Let us consider in details a laced compression member with two main components. The effective slenderness ratio of such a member is given by:

$$\lambda_{eff} = \frac{l}{i_{eff}} \qquad (7)$$

and the effective radius of gyration:

$$i_{eff} = \sqrt{\frac{I_{eff}}{A}},$$

$$I_{eff} = 2\left(I_f + 0.25A_f h_0^{\ 2}\right) = \\ = 0.5A_f h_0^{\ 2}\left[1 + 4\left(\frac{i_f}{h_0}\right)^2\right] \approx 0.5A_f h_0^{\ 2}$$

$$A = 2A_f, \qquad i_f = \sqrt{\frac{I_f}{A_f}} \qquad (9)$$

In equations (8) and (9), A_f, I_f are the cross sectional area and the moment of inertia of one chord, h_0 - the distance between the centres of gravity of member chord sections.

The shear stiffness of the laced column is given by:

$$S_v = \frac{EA_d a h_0^{\ 2}}{\eta l_d^{\ 3}}, \qquad l_d = \left(h_0^{\ 2} + a^2\right)^{1/2} \qquad (10)$$

where: l_d - length of the diagonal lacing, A_d - its cross sectional area, η - shear stiffness factor depending on the type of lacing system (for details see Table 1).

The second order moment at the mid-length section of the compound column yields from equation (3):

$$M_s = N_{Sd}v_{max} = \frac{N_{Sd}e_0}{1 - \dfrac{N_{Sd}}{N_{cr,red}}} \qquad (11)$$

The chord force at mid-length of the column should be determined from:

$$N_{f,Sd} = \frac{N_{Sd}}{2} + \frac{M_s}{h_0} = N_{Sd}\left(\frac{1}{2} + \frac{\dfrac{e_0}{h_0}}{1 - \dfrac{N_{Sd}}{N_{cr,red}}}\right) \qquad (12)$$

According to Eurocode 3, the chord force has to be checked against the chord buckling resistance as follows:

- chord nonslender section (class section 1 ,2 or 3):

$$\frac{N_{f,Sd}}{N_{fb,Rd}} \leq 1 \qquad (13)$$

- chord slender section (section of class 4):

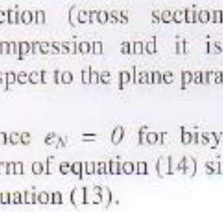
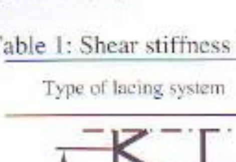
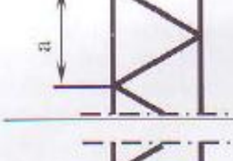
$$\frac{N_{f,Sd}}{N_{fb,Rd}} + \frac{N_{f,Sd} e_N}{M_{fc,Rd}}(1+\delta) \leq 1,$$

$$\delta = \frac{1.8\overline{\lambda}_f N_{f,Sd}}{N_{fb,R}} \leq 0.5 \qquad (14)$$

where: e_N - shift of the chord effective section centroidal axis with reference to the axis of nominal section (cross section is subjected to uniform compression and it is at least symmetrical with respect to the plane parallel to the lacing planes).

Since $e_N = 0$ for bisymmetric chord sections, the form of equation (14) simplifies in this case to that of equation (13).

Table 1: Shear stiffness factor

Type of lacing system	Factor η
	1
	$\frac{1}{2}$
	$\frac{1}{2}\left(1 + \frac{A_d h_0^3}{A_h l_d^3}\right)$ A_h- cross sectional area of the horizontal lacing

The buckling resistances of the chord shall be taken as

- characteristic value:

$$N_{fb,R} = \chi_f N_{fc,R} \qquad (15)$$

- design value:

$$N_{fb,Rd} = \chi_f N_{fc,Rd} \qquad (16)$$

The characteristic compression resistances of the cross section are given by

- for a chord nonslender section:

$$N_{fc,R} = A_f f_y \qquad (17)$$

- for a chord slender section:

$$N_{fc,R} = \beta_A A_f f_y \qquad (18)$$

and the design compression resistance

$$N_{fc,Rd} = \frac{N_{fc,R}}{\gamma_{M1}} \qquad (19)$$

where: $\beta_A = A_{f,eff} / A_f$ - effective area factor of class 4 section, f_y - nominal value of the steel yield stress, γ_{M1} - partial safety factor for resistance.

The effective cross section area is calculated as the area composed of walls plate segment effective widths times the nominal thickness of the wall. The effective width of the individual plate segment is calculated from the following equation

$$b_{eff} = \rho b \qquad (20)$$

where: ρ - the local buckling reduction factor obtained as follows

- when $\overline{\lambda}_p \leq 0.673 \quad \rho = 1 \qquad (21a)$

- when $\overline{\lambda}_p > 0.673 \quad \rho = \frac{\overline{\lambda}_p - 0.22}{\overline{\lambda}_p^{\,2}} \qquad (21b)$

The relative plate slenderness ratio is given by

$$\overline{\lambda}_p = \frac{b}{t}\frac{1}{28.4\varepsilon\sqrt{k_\sigma}}, \quad \varepsilon = \left(\frac{235}{f_y}\right)^{1/2} \qquad (22)$$

where: k_σ - local buckling factor that is equal to 4 for internal sectional plate elements (supported on two edges) and equal to 0.43 for outstand sectional plate elements (supported on one edge).

The effective width of outstand plate elements is measured from the supporting edge while for internal plate elements is divided by two and each half of the calculated effective width is measured from each supporting edge.

The buckling coefficient χ is the reduction factor for the buckling mode associated with flexural deformations parallel to the lacing planes

$$\chi = \frac{1}{\phi + \left(\phi^2 - \overline{\lambda}^2\right)^{1/2}},$$

$$\phi = \frac{1}{2}\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] \qquad (23)$$

where: α - imperfection parameter, $\overline{\lambda}$ - dimensionless slenderness.

In case of chords made of rolled sections, the imperfection parameter α shall be taken as follows

- for standard universal beam I-sections for which $h/b > 1.2$: $\alpha = 0.34$,
- for standard universal column I-sections for which $h/b \leq 1.2$ and also for [-sections: $\alpha = 0.49$.

For the design of chord sections, the calculation of the reduction factor $\chi = \chi_f$ is based on the dimensionless slenderness of the chord $\overline{\lambda}_f$. This slenderness ratio should be determined in the plane of lacing system, for the buckling length equal to the system length a between lacing connections:

- for a chord nonslender section:

$$\overline{\lambda_f} = \frac{\lambda_f}{\lambda_0} \quad (24)$$

- for a chord slender section:

$$\overline{\lambda_f} = \frac{\lambda_f}{\lambda_0}(\beta_A)^{1/2} \quad (25)$$

where the chord slenderness:

$$\lambda_f = \frac{a}{i_f} \quad (26)$$

and the slenderness ratio at first yielding

$$\lambda_0 = \pi\left(\frac{E}{f_y}\right)^{1/2} = 93.9\varepsilon \quad (27)$$

The design moment resistance of the cross section in equation (12) can be expressed as follows

$$M_{fc,Rd} = \frac{M_{fc,R}}{\gamma_{M1}} \quad (28)$$

where the characteristic moment resistances

- for a chord nonslender section:

$$M_{fc,R} = W_{f,el} f_y \quad (29)$$

- for a chord slender section:

$$M_{fc,R} = W_{f,eff} f_y \quad (30)$$

and: $W_{f,el}$ - elastic sectional modulus of the nominal cross section in the plane parallel to lacing system, $W_{f,eff}$ - elastic sectional modulus of the effective cross section in the same plane.

The lacing forces adjacent to the end panels should be derived from the internal shear force V_S taken as:

$$V_s = \frac{\pi M_s}{l} = \frac{\pi N_{Sd} e_0}{l\left(1 - \frac{N_{Sd}}{N_{cr,red}}\right)} = \frac{\pi N_{Sd}}{500\left(1 - \frac{N_{Sd}}{N_{cr,red}}\right)} \quad (31)$$

The force N_d in the diagonal lacing of the end panel is given by:

$$N_d = \frac{V_s l_d}{2h_0} = \frac{\pi N_{Sd}}{1000\left(1 - \frac{N_{Sd}}{N_{cr,red}}\right)} \frac{l_d}{h_0} \quad (32)$$

The size of the diagonal lacing shall satisfy the following design criterion

$$\frac{N_d}{N_{db,Rd}} \le 1 \quad (33)$$

The buckling resistance of the diagonal lacing $N_{db,Rd}$ is determined from equation (16), where the cross sectional area A_d is used and the buckling coefficient $\chi = \chi_d$ is calculated according to equation (23) with use of the slenderness ratio $\overline{\lambda_d}$. In case of a single angle lacing of a nonslender section, the dimensionless slenderness takes the form

$$\overline{\lambda_d} = \frac{\lambda_d}{\lambda_0} \quad (34)$$

where the slenderness of diagonal lacing

$$\lambda_d = \frac{l_d}{i_{d,\min}} \quad (35)$$

The imperfection parameter α for an angle lacing shell be taken as 0.49.

Using the design procedure recommended by Eurocode 3, one can calculate the maximum design load of a laced column for given chord section, lacing section, column length l and distance between chords h_o, as a function of the chord system length $a = l/n_0$, where $n_0 \ge 2$ is the integer number. For a chord nonslender section the maximum design load of the compound column yields from equation (13), after the substitution of equations (12) and (11)

$$N_{Sd}^{\ 2} - 2cN_{Sd} + 2c_0 = 0 \quad (36)$$

where

$$c_0 = \chi_f N_{fc,Rd} N_{cr,red},$$

$$c = \left(\frac{1}{2} + \frac{e_0}{h_0}\right) N_{cr,red} + \frac{c_0}{N_{cr,red}} \quad (37)$$

Solving the quadratic algebraic equation (37) yields

$$N_{Sd} = c - \sqrt{c^2 - 2c_0} \quad (38)$$

For chord slender monosymmetric sections, the calculations are more laborious since the maximum load has to be determined from the higher order algebraic equation [see equation (14)] and one has to resort to iterative methods.

To evaluate the maximum load on the laced member with reference to the different panel length (the distance between the lacing-to-chord connections), the following equation is recommended to ensure a technological ease of the fabrication of lacing connections

$$arctg\left(\frac{h_0}{a}\right) \ge 30^\circ, \qquad arctg\left(\frac{h_0}{a}\right) \le 60^\circ \quad (39)$$

3 PROPOSED METHOD OF IMPLICIT IMPERFECTION PARAMETER (EIP MODEL)

The proposed design procedure is based on the observation that the compound member is subjected to different buckling modes, which may be classified as local and global. The local modes are referred to instability of chord sectional walls (Fig. 3a) or to instability of chord between the lacing-to-chord joints (Fig. 3b). These modes do not produce postbuckling bending deflections of the compound member treated as a whole. A global mode is given in Fig. 3c, where the member is treated as a compound member with finite values of the flexural stiffness and the shear stiffness. It is obvious that in real design situations there is a coupling of local and global buckling modes, and that a separate treatment of these modes can lead to an unsafe design. Authors' proposal is therefore based on the treatment of all local buckling modes in the same way as that related to plate buckling of the sectional walls in Eurocode 3.

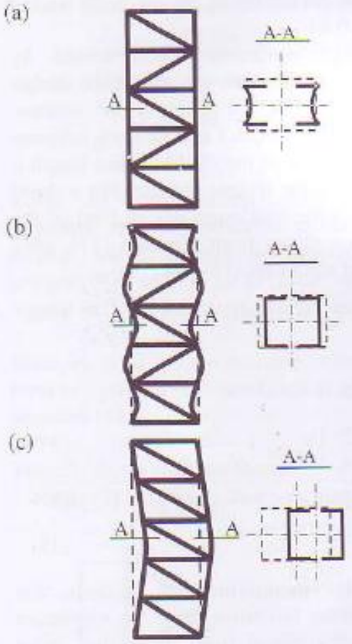


Figure 3: Possible buckling modes of a compound member: a) local buckling of a chord section wall, b) local buckling of the chord, c) global buckling of the compound member

Referring to equations (14) for a slender section chord, the compression resistance of the compound column section can be accounted for all local buckling modes of failure

$$N_{mc,Rd} = 2\beta_f N_{fb,Rd} = 2\beta_f \chi_f N_{fc,Rd} \quad (40)$$

where the coefficient β_f can be derived from the quadratic algebraic equation as follows

- for a slender section chord of monosymmetric section ($e_N > 0$)

$$\beta_f = \frac{1}{k}\left[-(1+k_0)+\sqrt{k_0\left[k_0+2(k+1)\right]+1}\right] \quad (41a)$$

- for a slender section chord of bisymmetric section ($e_N = 0$)

$$\beta_f = 1 \quad (41b)$$

and the coefficients k, k_0 are defined as

$$k = \frac{3{,}6\overline{\lambda_f}}{\gamma_{M1}}, \qquad k_0 = \frac{M_{fc,Rd}}{e_N N_{fb,Rd}} \quad (42)$$

Other variables have the same meaning as in equation (14).

The design load on a compound column can therefore be checked against the buckling resistance as follows

$$\frac{N_{Sd}}{N_{mb,Rd}} \le 1 \quad (43)$$

where the buckling resistance of the compound column

$$N_{mb,Rd} = \chi_m N_{mc,Rd} \quad (44)$$

In equation (44), χ_m is the buckling coefficient χ to be calculated according (23) for the slenderness ratio

$$\overline{\lambda_m} = \frac{\lambda_m}{\lambda_0}\left(\beta_f \chi_f \beta_A\right)^{1/2} \quad (45)$$

and λ_m is the compound column slenderness ratio derived from Timoshenko theory of elastic stability of perfect columns of finite flexural and shear stiffnesses.

Referring to the differential equilibrium equation (1), putting $v_0=0$ and solving for a basic case of simply supported member, one can get the following relationship

$$\lambda_m = \sqrt{\lambda_{eff}^{\,2} + \pi^2 \frac{EA}{S_V}} \quad (46)$$

For the laced column consisting of two main components

$$\lambda_m = \lambda_{eff}\gamma_V, \qquad \gamma_V = \sqrt{1+\left(\frac{\lambda_V}{\lambda_{eff}}\right)^2}, \quad (47)$$

$$\lambda_V = \pi \frac{l_d}{h_0}\sqrt{\frac{vA}{A_d}\frac{l_d}{a}}$$

where v - shear stiffness factor from Table 1 and λ_{eff} - slenderness of the chords according to (7).

The compound column critical load in presence of the finite shear stiffness of lacing system can be determined as

$$N_{cr,red} = \frac{\pi^2 EA}{\lambda_m^{\,2}} \quad (48)$$

The lacing system can be checked according to the procedure given by Eurocode 3. The force transmitted by the lacing system and given by (32) is to be checked against the buckling resistance of the diagonal lacing according to (33).

The proposed design criterion represents the buckling of the column as a whole. It is obvious that the compound column section is a quasi-closed one. The method of implicit imperfection parameter requires an introduction of the appropriate imperfection parameter. The parameter $\alpha = 0.21$ is suggested herein since it has been recommended for hot rolled hollow sections in Eurocode 3.

The proposal developed in this paper has several advantages:

- The design procedure is simple and treats the buckling effects in a uniform way, i.e. the local buckling modes are assumed to affect the compression resistance of the compound member cross section in the same way as the plate buckling of sectional wall segments.
- The buckling resistance of the compound member is the only criterion that needs to be checked regardless of whether the member chords are subjected to secondary bending due the shifting of the centre of gravity in the case of slender sections or not. The effect of secondary bending is taken care of by the reduction factor β_f. The effect of local buckling of sectional walls is taken care of by the reduction factor β_A and finally, the effect of local buckling of chord members between the lacing-to-chord connections – by the reduction factor χ_f.
- The imperfections are taken into account in an implicit way. The residual stresses and the initial crookedness of the chord between the lacing connections are taken into consideration in the reduction factor χ_f while the initial crookedness of the compound member – in the reduction factor χ_m.

4 COMPARATIVE ANALYSIS

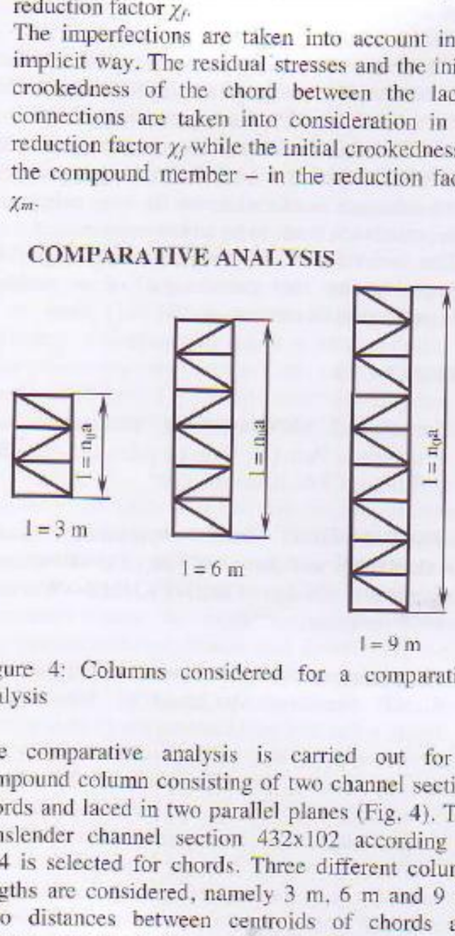


Figure 4: Columns considered for a comparative analysis

The comparative analysis is carried out for a compound column consisting of two channel section chords and laced in two parallel planes (Fig. 4). The nonslender channel section 432x102 according to BS4 is selected for chords. Three different column lengths are considered, namely 3 m, 6 m and 9 m. Two distances between centroids of chords are chosen, namely 0.404 and 0.604. The number of lacing system panels considered for each length of the column and each distance between the chord centroids satisfies the technological criteria (39). The steel grade Fe 360 is assumed according to Eurocode 3. The nominal yield strength is taken as f_y = 235 MPa (ε = 1) since the maximum thickness of sectional wall segments is not greater than 40 mm. The partial safety factor is equal to 1.1.

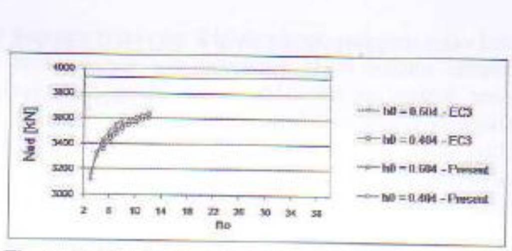


Figure 5: Maximum column design loads N_{Sd} for l = 3 m

The maximum design load N_{Sd} on the compound column is calculated according to Eurocode 3 model and according to the model of present study. The results are presented in Fig. 5 for the column of 3 m in length, Fig. 6 – for the column of 6 m in length and finally Fig. 7 – for the column of 9 m in length. The maximum loads corresponding to flexural buckling of columns in the direction perpendicular to the lacing system are also drawn in all the figures as horizontal lines. They are equal to: 3924.5 kN - for the column of 3 m in length, 3526.2 kN - for the column of 6 m in length and 3085.4 kN - for the column length of 9 m in length.

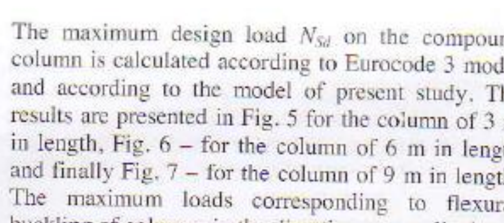


Figure 6: Maximum column design loads N_{Sd} for l = 6 m

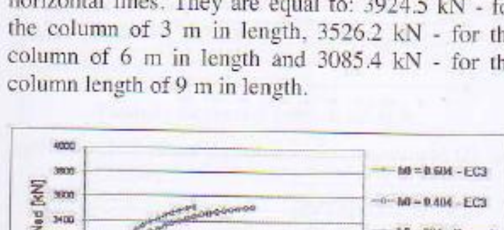


Figure 7: Maximum column design loads N_{Sd} for l = 9 m

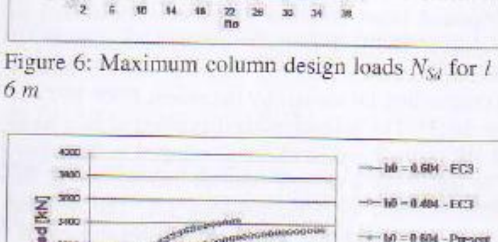


Figure 8: Maximum column design loads V_S for l = 3 m

The shear forces V_S corresponding to the design load N_{Sd} are given in Fig. 8 for the column of 3 m in length, Fig. 9 – for the column of 6 m in length and finally Fig. 10 – for the column of 9 m in length. The load corresponding to the 0.01% of the compound column squash load, conventionally suggested in some design specifications as the shear force for design of lacing members [2], is equal to: 39.2 kN. This load is much higher than those computed according to the Eurocode 3 model and the model of present study.

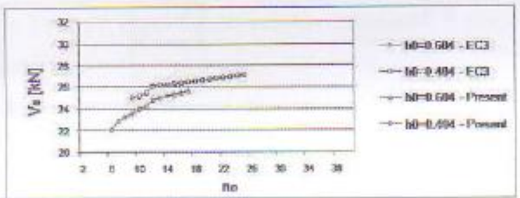


Figure 9: Maximum column design loads V_S for $l = 6m$

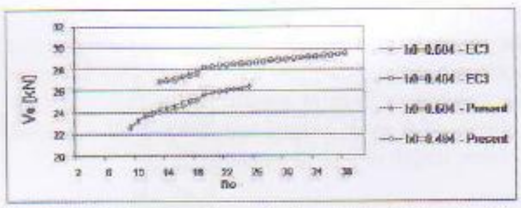


Fig. 10 Maximum column design loads V_S for $l = 9m$

5 CONCLUSIONS

Two methods for the design procedure of steel compound laced columns have been considered in the paper. The first method (a reference method) is referred here as to the EIP method. This method is recommended for design by the recent ENV 1993-1-1 code [1]. The second method is referred here as to the IIP method and has been developed in the paper. It is based on a design concept introduced to the Polish national steel design code [2]. The detailed procedures for two methods have been explained and programmed in a spreadsheet format of Microsoft Excel. Comparative analysis has been performed.
From the analysis carried out the following conclusions can be drawn:

- The method developed in the paper constitutes a simple and a handy alternative for design procedures recommended by current design specifications.
- Simplicity of the proposed method does not affect its accuracy; it is in a very good agreement with a more laborious method based on the reference model proposed in ENV 1993-1-1 [1].
- The proposed method gives the values of the resistance of compound columns within the range up to ±2% compared with those from the reference model; for the short columns, the results are slightly lower than those from the Eurocode 3 model while for the long columns – the resistance tends to be a bit overestimated.
- The flexural buckling mode out of the lacing planes may govern the design only if a long column is considered (see Fig. 7); otherwise the flexural buckling mode in the lacing planes has to be considered in the design criterion see Figs. 5 and 6).
- The proposed method gives the values of shear forces for the design of lacing members within the range up to ±2% if compared with those from the reference model; for the short columns, the results are placed a bit below those gotten from the reference model while for the long columns – the resistance tends to be a bit overestimated.
- The accuracy of the proposed model can be improved by the introduction of a variable imperfection factor α.

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